Dr Demetres Christofides

PhD Position in combinatorics

The successful candidate will work with me in the area of combinatorics and more particularly in the areas of extremal combinatorics, extremal graph theory and/or random graphs. Familiarity with some of these areas at a master’s level is desirable. The plan for the Ph.D. student is to start in one of the following projects. The choice of project will be done depending on the student’s interests and previous knowledge.

Random Cayley graphs
The theory of random graphs is very well studied (see e.g. [2]). In the classical model \( G(n, p) \) of random graphs, we start with \( n \) vertices and add each possible edge between the vertices independently at random with probability \( p \). In the random Cayley graph model \( G(G, p) \) we start with a group \( G \), choose a random subset \( S \) of \( G \) where each element is chosen independently with probability \( p \) and then construct the Cayley graph of \( G \) with respect to \( S \). The aim of this project is to translate some of the classical results of \( G(n, p) \) to the Cayley setting. Some work in this direction has already been done in [5, 6, 7].

Hamilton cycles in graphs
The project here is related to the famous conjecture of Lovász [9] which states that every connected vertex-transitive graph has a Hamilton path. Despite the vast literature that was generated from this problem (see e.g. [8] and its references) we are still very far from resolving this conjecture.
Here, we propose to study some problems emanating from [3] in which we managed to prove the conjecture for the case of sufficiently large dense graphs.

Guessing numbers of graphs
A graph guessing game is played on a graph (i.e. a finite set of vertices together with a set of edges between some pairs of vertices). There is one player for each vertex of the graph and they all simultaneously toss a coin. Each player can see the outcomes of the coin tosses of all of its neighbours but not its own outcome. Then they all have to simultaneously guess the outcome of their own coin toss and the game is won if they all guess correctly. It turns out that the extra information that the players have can help to improve their winning probability. This improvement depends on the underlying graph and is called its (asymptotic) guessing number.
In [4] we determined the guessing number of a large family of graphs. We also put forward a very natural conjecture which we recently disproved in [1]. This shows that we don’t yet have a very good understanding of guessing numbers. There is however a plethora of open problems that arise from [4, 1] and the purpose of this project is to work on some of them.

REFERENCES