

MODULE DESCRIPTOR

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| MODULE TITLE | NUMERICAL ANALYSIS | | |
| MODULE CODE | MA2852 (L5) | CREDIT VALUE | 20 CREDITS (10 ECTS) |
| CAMPUS | UCLAN CYPRUS | | |
| SCHOOL | SCHOOL OF SCIENCE | | |

MODULE AIMS

Modern physical scientists require knowledge of numerical methods, which are used to solve problems that may be analytically intractable. Mathematicians require a similar knowledge, but in their case this knowledge must include the background theory and not simply a tool-kit of methods; the theory relates especially to the inherent limitations of the numerical methods in question.

The aims of the module are to:

- Develop students' knowledge of numerical methods, with emphasis on solving algebraic equations, solving linear systems of equations, interpolation, and calculus.
- Build on students' existing knowledge of mathematical methods and analysis, to place numerical methods on a firm theoretical foundation.
- Inform students of the inherent errors and other potential problems associated with numerical analysis, and train students to analyse these errors.
- Develop students' confidence in both their analytical and computational skills.

MODULE CONTENT

The module will present a range of numerical methods, and theorems and proofs related to the convergence, error bounds, and stability of these methods.

Preliminaries

Analysis revision; Motivation for numerical analysis; Sources and types of errors in numerical analysis; Propagation of errors; Floating-point arithmetic; Well-posed and well-conditioned problems; Convergence and stability of numerical algorithms.

Solving Algebraic Equations

Bisection method; Newton's Method; The secant method; Fixed-point iteration; Convergence theorems and error bounds for these methods; Comparison of convergence and errors for different methods; Possible extension of these methods; Multiple roots, and why they change the convergence of the methods.

Interpolation

Piecewise linear interpolation; Weierstrass's approximation theorem; Existence and uniqueness of Lagrange interpolating polynomials; Lagrange interpolation (with error formula); Existence and uniqueness of Hermite interpolating polynomials; Hermite interpolation (with error formula); The trouble with interpolating polynomials (instability); Piecewise polynomial interpolation (with error formula); An introduction to spline interpolation.

Approximation theory

Discrete least-square approximation; Orthogonal polynomials; Chebishev polynomials; Rational function approximation; Trigonometric polynomial approximation.

Differentiation and Integration

Numerical differentiation, with error estimates; Richardson extrapolation for improved accuracy; Degree of precision in quadrature; The trapezoidal method; Simpson's method; Error bounds for these methods; Introduction to higher-order quadrature and the Newton-Cotes formulae; Composite methods.

Linear Algebra

Linear systems; Gaussian elimination; Pivoting schemes to avoid round-off errors etc; LU factorization; special types of matrices; Choleski method, Applications to Spline polynomials (Interpolation).

INTENDED LEARNING OUTCOMES

| On successful completion of this module a student will be able to: | |
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| 1. | Display basic knowledge of the ideas of numerical analysis, including the different types of errors which must be considered, and the convergence and stability of numerical methods. |
| 2. | Apply a range of methods, and compare their relative strengths and weaknesses, for problems including: solving algebraic equations, interpolating data, differentiating and integrating functions, and solving linear systems of equations. |
| 3. | Apply rigorous analytical methods to derive convergence theorems and error bounds for a wide range of numerical methods. |
| 4. | Develop computer programs, using suitable software, which implement a described numerical method. |

TEACHING METHODS

The module will be delivered through lectures where the course material is presented and worked examples given. To develop student skills, practical worksheets will be provided for the students to attempt and these will be discussed in the workshops.

The module will be assessed principally by examination. However, two coursework assignments will be set to give students practical experience throughout the year. Both of the assignments will consist of a selection of practical exercises, based on physical or mathematical problems, to solve with the aid of numerical software.

ASSESSMENT METHODS

The module is assessed through 2 Computer-based assignments and a written examination.